

Lepton Transmutation in the Dualized Standard Model

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Abstract

The successful explanation of fermion mixing and of the fermion mass hierarchy by the Dualized Standard Model (DSM) scheme is based on the premises of a fermion mass matrix rotating in generation space with changing scales at a certain speed, which could in principle lead to sizeable flavour-violation observable in high sensitivity experiments such as BaBar. However, a full perturbative calculation to 1-loop order reported here shows that this kinematical, flavour-violating effect of a rotating mass matrix is off-set in the DSM by parallel effects from rotating wave functions and vertices giving in the end only very small flavour-violations which are unlikely to be detectable by present experiments. The result means that at least for the present the DSM scheme has survived yet another threat to its validity, which is indeed its most stringent and dangerous to-date. It also provides some clarification of certain concepts connected with the rotating mass matrix which had previously been found puzzling.

1 Introduction

The Dualized Standard Model (DSM) [1] that we suggested has had some, to us, quite remarkable successes as a candidate solution to the fermion generation puzzle. Apart from offering a *raison d'être* for 3 generations of fermions, it gives a very simple explanation for such otherwise mysterious phenomena as fermion mixing and hierarchical fermion mass spectrum. Indeed, with only 3 adjustable parameters the DSM scheme is able to reproduce already at the one-loop level the following quantities all within present experimental limits: the 3 mass ratios m_c/m_t , m_s/m_b and m_μ/m_τ , all the 9 matrix elements $|V_{\alpha\beta}|$ of the CKM quark mixing matrix [2], plus the 2 elements $|U_{\mu 3}|$ and $|U_{e 3}|$ of the MNS lepton mixing matrix [3] measured in neutrino oscillation experiments [4, 5, 6]. This means in particular that the DSM scheme automatically reproduces an empirical fact which has recently caught much attention and been much wondered at, namely that the angle $|U_{\mu 3}|$ as found in atmospheric neutrinos [4, 5] is near maximal while the corresponding angles $|V_{ts}|$ and $|V_{cb}|$ for quarks are very small. Furthermore, even for the other measured mass and mixing parameters, namely m_u, m_d, m_e , and $|U_{e 2}|$, which are beyond the reach of the one-loop calculation so far performed, the estimates obtained by extrapolation are nevertheless quite sensible. Altogether, the above quantities account for 12 of the Standard Model's twenty-odd parameters which have no explanation in the conventional formulation but are simply introduced as empirical inputs. To us, this much agreement with experiment obtained with only 3 adjustable parameters appear nontrivial [7, 8].

However, all these apparent “successes” of the DSM rely on the scheme's prediction that the fermion mass matrix should change its orientation in generation space (rotate) with changing energy scale in a prescribed manner. That the fermion mass matrix should rotate with changing scale is not in itself special to the DSM, since even in the conventional formulation of the Standard Model the mass matrix will rotate by virtue of the renormalization group equation [9] so long as there is nontrivial mixing between the up and down fermion states [10]. But the speed of rotation so obtained is far below that required to derive the DSM results on fermion mass hierarchy and mixing summarized in the preceding paragraph [11]. Now, requiring a rotation of the mass matrix at a high speed could be dangerous for it could lead to sizeable flavour-violation in contradiction to experiment. For instance, a rotating lepton mass matrix means that the lepton flavour states which are defined to be diagonal states of the mass matrix at some prescribed scale(s) will in general no longer remain diagonal states at a different energy scale,

either of the mass matrix itself or of reaction amplitudes which depend on it. Hence, as suggested in [11], in certain reactions, leptons can change their flavours (transmute), simply by virtue of the kinematics of a rotating mass matrix even in the absence of a flavour-changing vertex. The amount of flavour-violation so induced depends on the speed at which the mass matrix rotates. Specifically, according to [12], a lepton mass matrix rotating at the speed demanded by DSM to give the results listed in the last paragraph could lead via kinematics alone to a cross section for the transmutation reaction: $e^+e^- \rightarrow \mu^\pm\tau^\mp$ of as much as 80 fb at 10.58 GeV, where BaBar, for example, has collected already 20 fb⁻¹ of data after only a year of running. In other words, such a phenomenon, if real, could readily be observed in principle by an analysis of the existing BaBar data. Hence, to test the validity of the DSM scheme, it is imperative to ascertain whether such effects are indeed obtained as predictions of the scheme.

The cited conclusion in [12], however, does not mean that the DSM will necessarily lead to lepton flavour violations of such magnitude. The calculation reported there evaluated only the kinematic effect of a rotating mass matrix considered in isolation without taking account of the dynamical mechanism driving it. In the DSM scheme, however, the rotating mass matrix is deduced as a consequence of a specific driving mechanism due to radiative corrections and this may give rise to other rotation effects which may modify or even cancel the effect calculated in [12] from the rotating mass matrix. Hence, to calculate properly in the DSM scheme the transmutation effects in say e^+e^- collisions, one should evaluate for consistency all radiative corrections to the amplitude to the same order. In particular, to 1-loop order, one should include one-loop insertions not just in the fermion propagator to obtain the rotating mass matrix, but also in the external fermion lines (wave functions) and the interaction vertices.

The purpose of this paper is, therefore, to perform a full perturbative calculation to one-loop order for lepton transmutation in γe and e^+e^- collisions in parallel to the calculations done in [13, 12]. We shall show that within the DSM framework as it is at present formulated, such a calculation can be unambiguously performed, all the relevant parameters in the scheme having already been determined by fitting the single-particle properties as detailed in the first paragraph. As a result of this calculation, we shall find that there are indeed scale-dependent rotation effects other than those deduced in [13, 12] from the rotating mass matrix alone, and that these rotation effects cancel exactly in 1-loop order, giving thus in total no scale-dependent transmutation. In other words, despite the requirement in the DSM scheme of

a sizeable rotation speed for the mass matrix in order to explain fermion mass hierarchy and mixing, no abnormally large flavour-violation is predicted. The flavour-violating effects obtained from renormalization at 1-loop level are of the order s/M^2 , where s is the interaction energy and M is the generic mass of the flavour bosons exchanged, and could thus be taken together with normal flavour-changing neutral current (FCNC) effects which are suppressed if M is large, as is generally supposed to be the case. Indeed, in an earlier investigation on FCNC effects in the DSM framework [14], M was estimated to be of order 500 TeV which would give very small flavour-violations at energies available to present experiments. It means therefore that the effect as considered in [11, 13], dangerous as it seemed at first sight, is unlikely to cause problem for the DSM.

The conclusion of the present calculation also helps in elucidating some basic but previously unclear concepts connected with fermion mixing and neutrino oscillations in the circumstances when the mass matrix rotates, which elucidation has wider connotations beyond the context of the present paper.

2 Preliminaries

The DSM scheme makes use of a theoretical result previously derived that there is in Yang–Mills theory a nonabelian version of electric–magnetic duality [15], namely that dual to the gauge group $SU(N)$, there is another local group $\widetilde{SU}(N)$ under which the theory is also symmetric, the potential of the latter group being related to the potential of the original gauge group via a nonabelian dual transform formulated in loop space which generalizes the Hodge star for the abelian theory. When the theory is quantized, it was shown that the dual group $\widetilde{SU}(N)$ is broken when the original gauge group $SU(N)$ is confined [16, 17]. Hence, in the case of colour as in standard chromodynamics, there is, dual to colour $SU(3)$, automatically a broken 3-fold symmetry $\widetilde{SU}(3)$ which can play the role of a “horizontal group” [18] for exactly 3 generations of fermions. Furthermore, the framework offers natural candidates for the Higgs fields breaking the generation symmetry in the form of frame-vectors (complex dreibeins) in $\widetilde{SU}(3)$ space, suggesting thereby the manner in which the generation symmetry is broken. The result is a highly predictive scheme for the description of fermion generations [19].

Although in themselves conceptually interesting with perhaps much wider implications, the assertions of the preceding paragraph concern us here only

in yielding a particular form of the Yukawa coupling and of the Higgs potential. The suggested Yukawa coupling between Higgs and fermion fields takes the form:

$$\sum_{(a)[b]} Y_{[b]} \bar{\psi}_L^a \phi_a^{(a)} \psi_R^{[b]} + h.c., \quad (1)$$

where $\psi_L^a, a = 1, 2, 3$ is the left-handed fermion field appearing as a dual colour triplet, and $\psi_R^{([b])}$ are 3 right-handed fermion fields, each appearing as a dual colour singlet. These are coupled to 3 triplets of Higgs fields $\phi_a^{(a)}, (a) = 1, 2, 3, a = 1, 2, 3$ each being a space-time scalar. Further, it was suggested that the Higgs fields themselves self-interact via the following potential:

$$V[\phi] = -\mu \sum_{(a)} |\phi^{(a)}|^2 + \lambda \left\{ \sum_{(a)} |\phi^{(a)}|^2 \right\}^2 + \kappa \sum_{(a) \neq (b)} |\bar{\phi}^{(a)} \cdot \phi^{(b)}|^2, \quad (2)$$

for which a general vacuum can be expressed as:

$$\phi^{(1)} = \zeta \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}; \quad \phi^{(2)} = \zeta \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}; \quad \phi^{(3)} = \zeta \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \quad (3)$$

with

$$\zeta = \sqrt{\mu/2\lambda}, \quad (4)$$

and x, y, z all real and positive, satisfying:

$$x^2 + y^2 + z^2 = 1. \quad (5)$$

Such a vacuum breaks the permutation symmetry of the ϕ 's which is maintained in both (1) and (2), and also the $\widetilde{U(3)}$ gauge symmetry completely making thus all the vector gauge bosons in the theory massive by eating up all but 9 of the original 18 Higgs modes.

As a result, the tree-level mass matrix for each of the 4 fermion-species T (i.e. whether of U - or D -type quarks, or of charged leptons L or neutrinos N) is of the following form:

$$m = \tilde{m} \frac{1}{2} (1 + \gamma_5) + \tilde{m}^\dagger \frac{1}{2} (1 - \gamma_5), \quad (6)$$

where \tilde{m} is a factorized matrix:

$$\tilde{m} \propto \begin{pmatrix} x \\ y \\ z \end{pmatrix} (a, b, c), \quad (7)$$

with a, b, c being the Yukawa couplings $Y_{[b]}$. By suitably redefining the right-handed fermion fields [20] which in no way affects the physics, one can rewrite the mass matrix in the more convenient form with no dependence on γ_5 :

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), \quad (8)$$

which is the form that we shall use in our calculations throughout. We note that this is of rank 1, having only one nonzero eigenvalue with eigenvector (x, y, z) the components of which, being Higgs vev's, are independent of the fermion-species T . Hence we have at the tree-level (i) that the fermion mass spectrum is 'hierarchical' with one generation much heavier than the other two, (ii) that the CKM matrix giving the relative orientation between the eigenvectors of the up- and down-type fermions is the identity matrix.

The results on fermion mass ratios and mixing parameters summarized at the beginning were obtained by the insertion of one (dual colour) Higgs loop to the fermion propagator as depicted in Figure 1(a). It has the effect

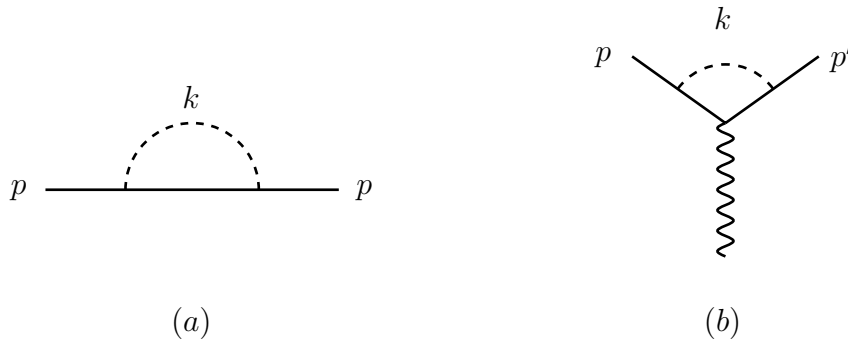


Figure 1: The relevant self-energy and vertex insertions

of rotating the mass matrix (8) with changing scale, hence giving nontrivial

mixing between up and down states and nonvanishing masses to the lower generation fermion. In order to calculate to the same loop order reaction processes by making all possible insertions, we shall need the explicit mass spectrum of the Higgs fields and their couplings to the fermions, which are deducible from (2) and (1) respectively.

Following Weinberg [20] we shall work in a real representation where the remaining 9 Higgs bosons were found earlier [21] to have at tree-level the following mass values:

$$\begin{aligned}
K = 1 : & \quad 8\lambda\zeta^2(x^2 + y^2 + z^2), \\
K = 2 : & \quad 4\kappa\zeta^2(y^2 + z^2), \\
K = 3 : & \quad 4\kappa\zeta^2(y^2 + z^2), \\
K = 4 : & \quad 4\kappa\zeta^2(z^2 + x^2), \\
K = 5 : & \quad 4\kappa\zeta^2(z^2 + x^2), \\
K = 6 : & \quad 4\kappa\zeta^2(x^2 + y^2), \\
K = 7 : & \quad 4\kappa\zeta^2(x^2 + y^2), \\
K = 8 : & \quad 0, \\
K = 9 : & \quad 0,
\end{aligned} \tag{9}$$

and the following couplings to fermions:

$$\bar{\Gamma}_K = \bar{\gamma}_K \frac{1}{2}(1 + \gamma_5) + \bar{\gamma}_K^\dagger \frac{1}{2}(1 - \gamma_5), \tag{10}$$

where

$$\bar{\gamma}_K = \rho |v_K\rangle \langle v_1|, \tag{11}$$

with ρ the strength of the Yukawa coupling, and

$$\begin{aligned}
|v_1\rangle &= \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \\
|v_2\rangle &= \frac{1}{\sqrt{y^2 + z^2}} \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}, \\
|v_3\rangle &= \frac{i}{\sqrt{y^2 + z^2}} \begin{pmatrix} 0 \\ y \\ -z \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
|v_4\rangle &= \frac{1}{\sqrt{z^2 + x^2}} \begin{pmatrix} x \\ 0 \\ z \end{pmatrix}, \\
|v_5\rangle &= \frac{i}{\sqrt{z^2 + x^2}} \begin{pmatrix} -x \\ 0 \\ z \end{pmatrix}, \\
|v_6\rangle &= \frac{1}{\sqrt{x^2 + y^2}} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}, \\
|v_7\rangle &= \frac{i}{\sqrt{x^2 + y^2}} \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix},
\end{aligned} \tag{12}$$

while the two remaining (degenerate) “zero modes” can be assigned the following vectors orthogonal to $|v_1\rangle$:

$$|v_8\rangle = -\beta \begin{pmatrix} y - z \\ z - x \\ x - y \end{pmatrix}; |v_9\rangle = \beta \begin{pmatrix} 1 - x(x + y + z) \\ 1 - y(x + y + z) \\ 1 - z(x + y + z) \end{pmatrix}, \tag{13}$$

with

$$\beta^{-2} = 3 - (x + y + z)^2. \tag{14}$$

These “zero modes” arise only by virtue of an “accidental” symmetry of the vacuum which is not present in the action itself and are thus unlikely to remain massless under radiative correction.

With the above information, one can now proceed to evaluate transmutation effects of scale-dependent rotation from 1-loop corrections to e^+e^- and γe reactions of present interest. The main diagrams to be evaluated are listed respectively in Figures 2 and 3, plus the corresponding crossed diagrams, where we notice that since the photon carries no generation (dual colour) index, it does not couple to the (dual colour) Higgs fields, so that only 2 types of loop insertions occur, namely either self-energy insertions in the fermion line (internal or external) or insertions in the fermion-fermion-photon vertex, as depicted in Figure 1. Although the calculations for these insertions are fairly standard, the fact that m is a rotating matrix does cause some complications which require consideration with due care. We shall examine these 2 types of insertions in turn.¹

¹For notation we follow closely [22]; for matrix complications and other intricacies see e.g. [20, 21].

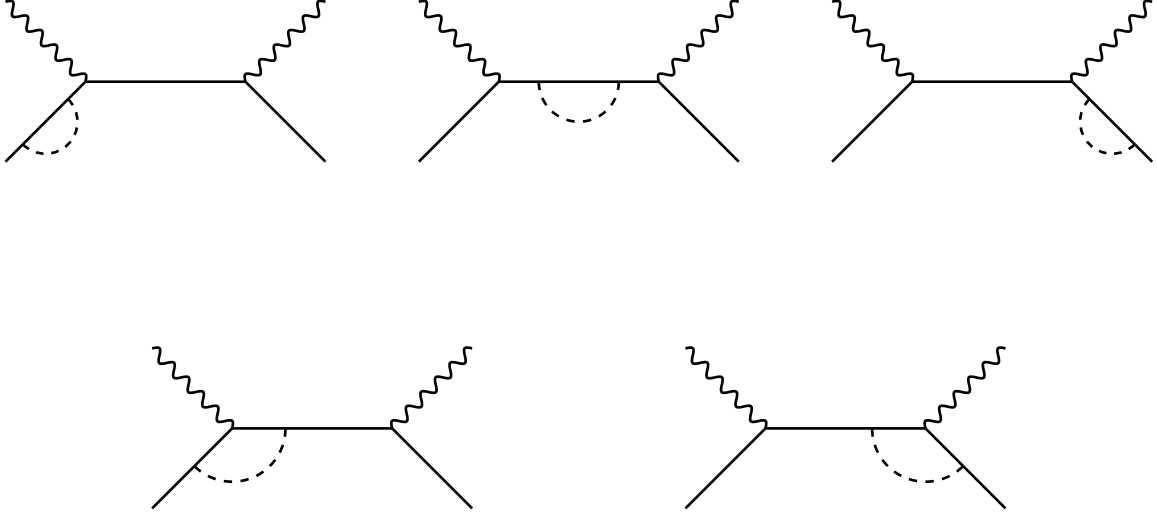


Figure 2: 1-loop insertions in the amplitude for γe collision

3 The Self-Energy Insertion

The fermion self-energy insertion of Figure 1(a) takes the form:

$$\Sigma(p) = \frac{i}{(4\pi)^4} \sum_K \int d^4k \frac{1}{k^2 - M_K^2} \bar{\Gamma}_K \frac{(\not{p} - \not{k}) + m}{(p - k)^2 - m^2} \bar{\Gamma}_K, \quad (15)$$

with m and $\bar{\Gamma}_K$ given in (8) and (10). Combining denominators by the standard Feynman parametrization and shifting the origin of the k -integration as usual, one obtains:

$$\Sigma(p) = \frac{i}{(4\pi)^4} \sum_K \int_0^1 dx \bar{\Gamma}_K \left\{ \int d^4k \frac{\not{p}(1-x) + m}{[k^2 - Q^2]^2} \right\} \bar{\Gamma}_K, \quad (16)$$

with

$$Q^2 = m^2 x + M_K^2 (1-x) - p^2 x(1-x), \quad (17)$$

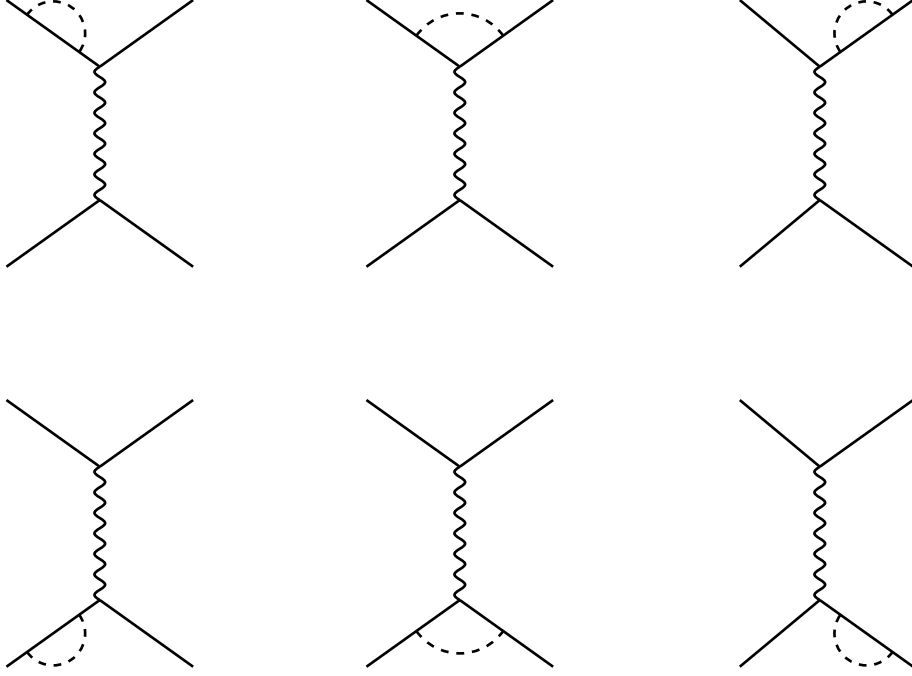


Figure 3: 1-loop insertions in the amplitude for e^+e^- collision

where we note that m , being a matrix in generation space, cannot be commuted through the coupling $\bar{\Gamma}_K$'s. The integration over k in (16) is divergent and has to be regularized. Following the standard dimensional regularization procedure, one obtains:

$$\Sigma(p) = -\frac{1}{16\pi^2} \sum_K \int_0^1 dx \bar{\Gamma}_K \{ \bar{C} - \ln(Q^2/\mu^2) \} [\not{p}(1-x) + m] \bar{\Gamma}_K, \quad (18)$$

with \bar{C} being the divergent constant:

$$\bar{C} = \lim_{d \rightarrow 4} \left[\frac{1}{2 - d/2} - \gamma \right], \quad (19)$$

to be subtracted in the standard $\overline{\text{MS}}$ scheme.

To extract the renormalized mass matrix:

$$m' = m + \delta m \quad (20)$$

from $\Sigma(p)$, one normally puts in the denominator $p^2 = m^2$ and commutes \not{p} in the numerator to the left or right and replace by m [20]. However, m being now a matrix, this operation is a little more delicate. In order to maintain the “hermitan”, left-right symmetric form (6) for the renormalized mass matrix m' , we split the \not{p} term into two halves, commuting half to the left and half to the right before replacing by m , and hence obtain for δm the following:

$$\begin{aligned}
\delta m = & \frac{\rho^2}{16\pi^2} \sum_K \int_0^1 dx \{ \bar{\gamma}_K m [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) \\
& + \bar{\gamma}_K^\dagger m [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \} \\
& + \frac{\rho^2}{32\pi^2} \sum_K \int_0^1 dx (1-x) m \{ \bar{\gamma}_K^\dagger [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) \\
& + \bar{\gamma}_K [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \} \\
& + \frac{\rho^2}{32\pi^2} \sum_K \int_0^1 dx (1-x) \{ \bar{\gamma}_K [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2} (1 + \gamma_5) \\
& + \bar{\gamma}_K^\dagger [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K \frac{1}{2} (1 - \gamma_5) \} m,
\end{aligned} \tag{21}$$

with

$$Q_0^2 = Q^2|_{p^2=m^2} = m^2 x^2 + M_K^2 (1-x). \tag{22}$$

This is what was evaluated² in [21] and is all that is needed to calculate the single-particle properties such as fermion mass and mixing parameters of interest to us there. We notice in particular that the mass matrix m' after renormalization rotates with changing scale μ , which was the crucial property that gave rise in our earlier papers [21, 7] to the distinctive fermion mass and mixing patterns observed in experiment.

For investigating transmutation processes, however, more information is

²There is a sign error in the first term on the right of eq. (4.14) of [21] due to a misprint in the formula for $\Sigma^{(\phi^1)}$ in eq. (3.2) quoted from [20] which means that the coefficient of the last term in eq. (5.8) of [21] should be $3/(64\pi^2)$ instead of $5/(64\pi^2)$. Apart from the fact that the numerical values given for the parameter ρ in eq. (6.8) should be increased by a factor $\sqrt{5/3}$, no other result given in that paper or in its sequels such as [7] is affected by this error.

needed, for which purpose we write $\Sigma(p)$ as:

$$\Sigma(p) = -\frac{\delta m}{\rho^2} + \frac{1}{2}(\not{p} - m)B_L + \frac{1}{2}B_R(\not{p} - m) + \Sigma_c(p), \quad (23)$$

with

$$\begin{aligned} B_L &= -\frac{1}{16\pi^2} \sum_K \int_0^1 dx (1-x) \{ \bar{\gamma}_K^\dagger [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) \\ &\quad + \bar{\gamma}_K [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \}, \\ B_R &= -\frac{1}{16\pi^2} \sum_K \int_0^1 dx (1-x) \{ \bar{\gamma}_K [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2} (1 + \gamma_5) \\ &\quad + \bar{\gamma}_K^\dagger [\bar{C} - \ln(Q_0^2/\mu^2)] \bar{\gamma}_K \frac{1}{2} (1 - \gamma_5) \}, \end{aligned} \quad (24)$$

and

$$\begin{aligned} \Sigma_c(p) &= \frac{1}{16\pi^2} \sum_K \int_0^1 \{ \bar{\gamma}_K m \ln(Q^2/Q_0^2) \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) \\ &\quad + \bar{\gamma}_K^\dagger m \ln(Q^2/Q_0^2) \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \} \\ &\quad + \frac{1}{16\pi^2} \sum_K \int_0^1 (1-x) \{ \bar{\gamma}_K^\dagger \not{p} \ln(Q^2/Q_0^2) \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) \\ &\quad + \bar{\gamma}_K \not{p} \ln(Q^2/Q_0^2) \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \}. \end{aligned} \quad (25)$$

We note that Σ_c so extracted is both finite and independent of the renormalization scale μ . Indeed for those terms in the sum over K for which M_K is large, the contribution is only of order s/M_K^2 , as can be seen by writing:

$$\ln(Q^2/Q_0^2) = \ln \left[1 + \frac{(m^2 - p^2)x(1-x)}{m^2x^2 + M_K^2(1-x)} \right] \quad (26)$$

which for $0 \leq x \leq 1$ is, for large M_K , $\leq (m^2 - p^2)/M_K^2 \sim s/M_K^2$.

The insertion (23) when added to an internal fermion line thus gives:

$$\frac{1}{\not{p} - m} \longrightarrow \frac{1}{\not{p} - m'} - \frac{\rho^2}{2} B_L \frac{1}{\not{p} - m} - \frac{\rho^2}{2} \frac{1}{\not{p} - m} B_R - \rho^2 \frac{1}{\not{p} - m} \Sigma_c(p) \frac{1}{\not{p} - m}, \quad (27)$$

and when added to an external fermion line:

$$u(p) \longrightarrow u'(p) - \frac{\rho^2}{2} B_L u(p) - \rho^2 \frac{1}{\not{p} - m} \Sigma_c(p) u(p), \quad (28)$$

$$\bar{u}(p) \longrightarrow \bar{u}'(p) - \frac{\rho^2}{2} \bar{u}(p) B_R - \rho^2 \bar{u}(p) \Sigma_c(p) \frac{1}{\not{p} - m}, \quad (29)$$

where $u'(p)$ is a solution of the Dirac equation with the renormalized mass matrix m' :

$$(\not{p} - m') u'(p) = 0. \quad (30)$$

These conclusions follow closely those in e.g. ordinary QED apart from that, m being a matrix and therefore noncommuting, (i) B_L and B_R are different so that $u(p)$ and $\bar{u}(p)$ are renormalized differently, (ii) the finite part $\Sigma_c(p)$ applying on $u(p)$ or $\bar{u}(p)$ does not necessarily give zero. One notes also that $u(p)$ or $\bar{u}(p)$ picks up automatically just one-half of the B contribution, i.e. either $B_L/2$ or $B_R/2$, without the usual argument with adiabatic switching on and off of the interaction being invoked.

4 The Vertex Insertion

Next, the vertex insertion of Figure 1(b) takes the form:

$$\Lambda^\mu(p, p') = -\frac{i}{(2\pi)^4} \sum_K \int d^4 k \frac{1}{k^2 - M_K^2} \bar{\Gamma}_K \frac{(\not{p}' - \not{k}) + m}{(p' - k)^2 - m^2} \gamma^\mu \frac{(\not{p} - \not{k}) + m}{(p - k)^2 - m^2} \bar{\Gamma}_K. \quad (31)$$

Combining denominators as usual with the Feynman parametrization, then shifting the origin of the k -integration and dropping terms odd in k , one obtains:

$$\Lambda^\mu(p, p') = -\frac{2i}{(2\pi)^4} \sum_K \int_0^1 dx \int_0^x dy \int d^4 k \bar{\Gamma}_K \frac{\not{k} \gamma^\mu \not{k}}{[k^2 - P^2]^3} \bar{\Gamma}_K + \Lambda_c^\mu(p, p'), \quad (32)$$

with

$$\Lambda_c^\mu(p, p') = -\frac{2i}{(2\pi)^4} \sum_K \int_0^1 dx \int_0^x dy \int d^4 k \bar{\Gamma}_K \frac{\mathcal{N}}{[k^2 - P^2]^3} \bar{\Gamma}_K, \quad (33)$$

$$\mathcal{N} = [\not{p}'(1 - x + y) - \not{p}(1 - x) + m] \gamma^\mu [\not{p}x - \not{p}'(x - y) + m], \quad (34)$$

and

$$P^2 = m^2(1-y) + M_K^2 y - p^2 x(1-x) - p'^2(x-y)(1-x+y) + 2pp'(1-x)(x-y), \quad (35)$$

where $\Lambda_c^\mu(p, p')$, we note, is convergent, scale-independent, and of order s/M_K^2 for large M_K .

The divergent integral over k in (32) we regularize again by dimensional regularization obtaining an answer which we choose to write as:

$$\Lambda^\mu(p, p') = \frac{1}{2}\gamma^\mu L_L + \frac{1}{2}L_R\gamma^\mu + \Lambda_c^\mu(p, p'), \quad (36)$$

with

$$\begin{aligned} L_L &= -\frac{1}{16\pi^2} \sum_K \int_0^1 dx \int_0^x dy \{ \bar{\gamma}_K^\dagger [\bar{C} - \ln(P^2/\mu^2)] \bar{\gamma}_K \frac{1}{2}(1 + \gamma_5) \\ &\quad + \bar{\gamma}_K [\bar{C} - \ln(P^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2}(1 - \gamma_5) \}, \\ L_R &= -\frac{1}{16\pi^2} \sum_K \int_0^1 dx \int_0^x dy \{ \bar{\gamma}_K [\bar{C} - \ln(P^2/\mu^2)] \bar{\gamma}_K^\dagger \frac{1}{2}(1 + \gamma_5) \\ &\quad + \bar{\gamma}_K^\dagger [\bar{C} - \ln(P^2/\mu^2)] \bar{\gamma}_K \frac{1}{2}(1 - \gamma_5) \}, \end{aligned} \quad (37)$$

where \bar{C} is again the divergent constant in (19).

We notice that L_L and L_R in (37) are very similar to B_L and B_R in (24) obtained in the self-energy insertion. Indeed, if we take the difference $B_L - L_L$ ($B_R - L_R$), the divergent and scale dependent parts cancel leaving only a term proportional to:

$$\Delta I = \int_0^1 dx (1-x) \ln(Q_0^2) - \int_0^1 dx \int_0^x dy \ln(P^2), \quad (38)$$

which we shall show is of order s/M_K^2 for M_K large. This is not surprising since the self-energy and vertex insertions are related by the Ward identity:

$$\frac{\partial \Sigma(p)}{\partial p^\mu} = \Lambda^\mu(p, p), \quad (39)$$

which when applied to the formulae (23) and (36) would suggest such a result. To see this explicitly, we note first that for $s/M_K^2 \ll 1$, and $0 \leq x \leq 1, 0 \leq y \leq x$, one can approximate P^2 as:

$$P^2 \sim M_K^2 y - (p - p')^2 x(1-x) + m^2. \quad (40)$$

Secondly, we recall that the second integral on the right of (38) is only symbolic, because the expression

$$\int_0^x dy [\bar{C} - \ln(P^2)] \quad (41)$$

really means

$$\lim_{d \rightarrow 4} \int_0^x dy \Gamma(2 - d/2) (P^2)^{d/2-2}, \quad (42)$$

where the integral over y should be performed first before taking the limit for the proper regularization procedure, giving the estimate:

$$\bar{C}x - x \ln(M_K^2 x + m^2), \quad (43)$$

which when substituted in place of the integral over y in (38) is easily seen to give for (38) a value of order s/M_K^2 for large M_K as claimed.

With these observations for the vertex insertion in addition to those above for the self-energy insertion, we are now in a position to consider lepton-transmutations in e^+e^- and γe collisions.

5 Transmutation in γe and e^+e^-

By transmutation here, we mean a reaction which violates flavour-conservation by virtue of rotation effects (in generation space) under changes of scale which render the reaction amplitude non-diagonal in the flavour states. In the DSM scheme, this comes about mainly through loop diagrams with (dual colour) Higgs exchange. Indeed, it was the insertion of Figure 1(a) into the fermion propagator which gave rise to the rotating mass matrix in the first place and led to the DSM explanation of fermion mixing and the fermion mass hierarchy [21]. There were other insertions which gave mass matrix rotations, such as (dual colour) gauge boson loops and tadpoles, but these were shown to give only effects of much lower magnitude so as to be negligible for present purposes. In this paper, therefore, we shall be restricted to only 1-(dual colour)-Higgs-loop diagrams.

For the two reactions under consideration, one starts then with a mass matrix m diagonal in the lepton-flavour states τ, μ and e giving for the reaction:

$$\gamma \ell_\alpha \longrightarrow \gamma \ell_\beta, \quad (44)$$

at tree level the diagrams in Figure 4, and for the reaction:

$$e^+ e^- \longrightarrow \ell_\alpha^+ \ell_\beta^-, \quad (45)$$

the tree-level diagrams in Figure 5. Explicitly, the reaction amplitudes are, ignoring numerical factors:

$$\bar{u}(p') \gamma^\mu \frac{i}{(\not{p} + \not{k}) - m} \gamma_\mu u(p), \quad (46)$$

for the diagram of Figure 4(a), and:

$$[\bar{u}(p') \gamma^\mu u(p)] \frac{1}{(p' - p)^2} [\bar{v}(q) \gamma_\mu v(q')], \quad (47)$$

for the diagram of Figure 5(a), with similar formulae for the diagrams (b) in each case. The mass matrix m being diagonal in the flavour states, so also are the reaction amplitudes, giving thus no flavour-violation at this tree-level.

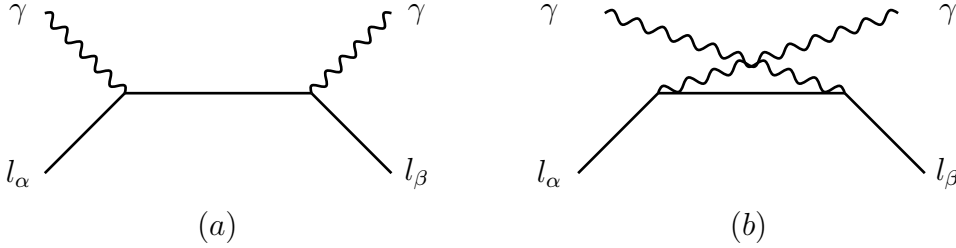


Figure 4: Tree diagrams for the reaction $\gamma \ell_\alpha \longrightarrow \gamma \ell_\beta$

To calculate now the amplitude for the reaction (44) to 1-loop order, we have to add to Figure 4 all 1-loop diagrams with (dual colour) Higgs exchange, i.e. the diagrams in Figure 2 with the fermion self-energy and vertex insertions studied in the preceding two sections, plus the diagram of Figure 6(a). This last-named diagram is finite and is easily seen to give only effects of order s/M_K^2 . Adding the other diagrams and making use of the results in (27), (29), and (36), one obtains to order ρ^2 the result:

$$\begin{aligned} & \bar{u}'(p') \gamma^\mu \frac{i}{(\not{p} + \not{k}) - m'} \gamma_\mu u'(p) \\ & + \frac{\rho^2}{2} \bar{u}(p') [\gamma^\mu (L_L - B_L) + (L_R - B_R) \gamma^\mu] \frac{i}{(\not{p} + \not{k}) - m} \gamma_\mu u(p) \\ & + \frac{\rho^2}{2} \bar{u}(p') \gamma^\mu \frac{i}{(\not{p} + \not{k}) - m} [\gamma_\mu (L_L - B_L) + (L_R - B_R) \gamma_\mu] u(p) \end{aligned} \quad (48)$$

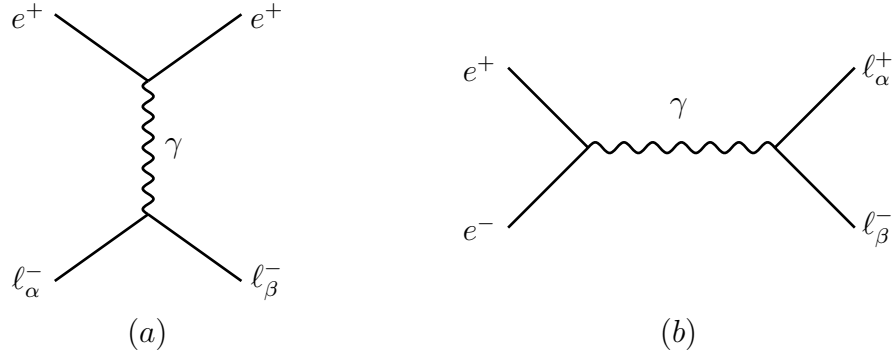


Figure 5: Tree diagrams for the reaction $e^+e^- \longrightarrow \ell_\alpha^+ \ell_\beta^-$

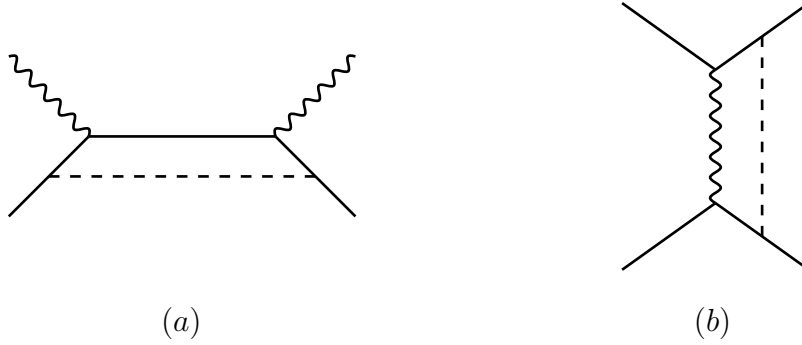


Figure 6: Convergent 1-loop diagrams

plus terms involving the quantities Σ_c and Λ_c^μ which we recall are of order s/M_K^2 for large M_K . We recall further that in the differences $B_L - L_L$ and $B_R - L_R$, the divergent and the scale dependent parts both cancel, leaving in each only a finite part which is again of order s/M_K^2 for large M_K . Hence, if M_K is indeed large for all K , then the renormalized amplitude (48) will reduce simply to the first term there.

However, M_K is large not for every K at tree-level where, as can be seen in (9), there are two modes $K = 8, 9$ with zero mass. Although these modes are expected eventually to acquire masses also from radiative corrections, they may need special consideration. Fortunately, it turns out that the contributions of these modes 8 and 9 to the amplitude (48) is diagonal in the flavour-states τ, μ, e and gives therefore no transmutation effects of present

interest. That this is so can be seen as follows. We note that in the various terms of the amplitude (48), factors of the following form repeatedly appear:

$$\begin{aligned}\bar{\gamma}_K \bar{\gamma}_K &= \rho^2 |v_K\rangle \langle v_1 | v_K\rangle \langle v_1|; & \bar{\gamma}_K^\dagger \bar{\gamma}_K^\dagger &= \rho^2 |v_1\rangle \langle v_K | v_1\rangle \langle v_K|; \\ \bar{\gamma}_K \bar{\gamma}_K^\dagger &= \rho^2 |v_K\rangle \langle v_1 | v_1\rangle \langle v_K|; & \bar{\gamma}_K^\dagger \bar{\gamma}_K &= \rho^2 |v_1\rangle \langle v_K | v_K\rangle \langle v_1|.\end{aligned}\quad (49)$$

For $K = 8, 9$, the vectors v_K are orthogonal to v_1 so that the two factors in the first row give zero, while those in the second row are diagonal in the basis v_1, v_8, v_9 . And since v_1 is by definition the vector for the heaviest state τ while v_8 and v_9 are orthonormal linear combinations of the vectors of μ and e , this means that when summed over $8, 9$, the factors in (49) are all either zero or diagonal in the flavour states τ, μ, e . Hence it follows that if one is interested only in non-diagonal processes as we are in this paper, then one can omit the terms for $K = 8, 9$ in the sums over K above.

One concludes therefore that to order s/M_K^2 , off-diagonal (i.e. flavour-violating) amplitudes for the reaction (44) is given just by the first term in (48), namely:

$$\bar{u}'(p') \gamma^\mu \frac{i}{(\not{p} + \not{k}) - m'} \gamma_\mu u'(p) \quad (50)$$

where we recall that m' is the renormalized mass matrix which rotates with changing scales so that the fermion propagator which depends on m' is no longer diagonal in the lepton-flavour states τ, μ, e at the energy scale where the reaction is measured. However, according to (50), when evaluating the amplitude, one is to sandwich this propagator not between the original lepton-flavour states $u(p)$ but between the renormalized states $u'(p)$, which are solutions of the equation (30) and are thus themselves eigenstates of the renormalized (rotated) mass matrix m' . The fermion propagator is thus diagonal between these states and give, to order s/M_K^2 , no non-diagonal matrix elements. This is in stark contrast to the effect estimated in [13] just from the kinematics of the rotating mass matrix which were quite sizeable. In a nutshell, this is because the renormalization mechanism in DSM scheme which drives the mass matrix rotation induces at the same time rotations in the fermion wave functions and in the interaction vertices, which neatly compensate one another to a good approximation to give in the end a near null effect.

Similar arguments applied to the reaction (45) lead to a similar conclusion. Of the 1-loop diagrams with (dual colour) Higgs exchange, the diagram of Figure 6(b) is finite and of order s/M_K^2 . Then, adding to the tree amplitude (47) the 1-loop diagrams of Figure 3 replaces the first factor in (47)

by:

$$\begin{aligned}
[\bar{u}(p')\gamma^\mu u(p)] \longrightarrow & [\bar{u}'(p')\gamma^\mu u'(p)] + \frac{\rho_c^2}{2}[\bar{u}(p')\gamma^\mu(L_L - B_L)u(p)] \\
& + \frac{\rho_c^2}{2}[\bar{u}(p')(L_R - B_R)\gamma^\mu u(p)]
\end{aligned} \tag{51}$$

plus terms involving Σ_c and Λ_c^μ . Again for off-diagonal (i.e. flavour-violating) elements, $L_L(L_R)$ cancels with $B_L(B_R)$ and Σ_c and Λ_c^μ are of order s/M_K^2 , leaving only the first term in (51) which has no off-diagonal elements. The same arguments hold for the last factor in (47) corresponding to the bottom half of the diagrams in Figure 3. Hence the conclusion is again that the DSM scheme predicts no flavour-violation in the reaction (45) up to terms of order s/M_K^2 .

Recalling our designation at the beginning of the section of transmutation as flavour-violation due to the rotational effects under changing scales, we thus conclude that the DSM scheme predicts no transmutation as such, at least at the 1-loop level so far investigated. And this is the case in spite of the rather fast rotation rate the scheme requires for the mass matrix in order to explain the observed fermion mixing and mass hierarchy.

There are flavour-violating effects, though not due to rotation, of the order of s/M_K^2 . But terms of this order would arise in any case, at least for (45), from the direct exchange of (dual colour) gauge and Higgs bosons. These (FCNC-type) effects would be common to any model in which fermion generations are interpreted as a gauged “horizontal” symmetry, and not specific to the DSM alone. The flavour-violation in such a context is generally taken to be suppressed by large masses for the exchanged bosons which can be estimated from the experimental bounds on flavour-violation. Specifically, a detailed analysis within the DSM scheme of meson mass differences and rare meson decays [14] and of $\mu - e$ conversion in nuclei [23] led to an estimate of the gauge boson mass μ_N of the order $\mu_N/\tilde{g} > 500$ TeV with \tilde{g} being the coupling, which is fairly typical for models with “horizontal” symmetries. Although no similar analysis has yet been performed for the Higgs exchange, a bound of an analogous order of magnitude is expected for the Higgs mass M_K appearing above, so that at the energy of ~ 10 GeV of present high sensitivity experiments, the anticipated flavour-violation, in (45) for example, will be very small. Alternatively, one can turn the considerations around and use the reaction (45) to set a bound on the gauge and Higgs boson masses. Although the bounds so deduced are probably going to be less stringent than those obtained from meson mass differences and rare meson decays even with

the data from new high sensitivity experiments, they will have the virtue of being free from the many crude assumptions made on the hadron physics inherent in the derivation of the bounds with the other methods.

6 Generalization to Other Cases

The conclusions on transmutation given in the preceding section have been deduced explicitly only in the specific DSM scheme as detailed in [21] and applied to two special reactions. However, the arguments involved such as the Ward identity relating wave function and vertex renormalization seem quite general and suggest that the result may hold in a more general context. The results presented above are thus likely to survive some changes in the details such as the Higgs spectrum of the DSM scheme as given, for example, in [21, 7]. This possibility is relevant for although the DSM as specifically given in [21, 7] has so far been remarkably successful in reproducing mass and mixing patterns, there may come a point when under further detailed examination, minor modifications become necessary for consistency either within the scheme itself or with experimental data such as, say, in CP-violation for which the scheme at present says nothing.

The conclusion above of no transmutation for DSM up to terms of orders s/M_K^2 may also hold for processes other than those two explicitly investigated, i.e. (44) and (45), which are given by 1-photon exchange. Although explicit calculations have yet to be performed to demonstrate that this is indeed the case, we wish now to explore some implications of such a generalization which seem to clarify certain concepts we have previously found puzzling. These conceptual questions would arise in any theory with a rotating mass matrix, not just in the DSM scheme alone.

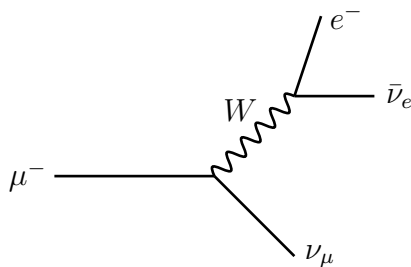


Figure 7: Decay of a μ giving ν_μ

A prime example of the sort of questions we wish to pose concerns the oscillation of atmospheric neutrinos as reported in [4, 5]. Here, what is supposed to have happened is that, say, from μ decay via the process depicted in Figure 7, one obtains a ν_μ which is not one of the neutrino mass eigenstates $\nu_i, i = 1, 2, 3$ but a linear combination of them, with each having a different mass and therefore propagating with a different wave length. Hence after a while, this initial ν_μ will no longer remain in a ν_μ state but become a linear combination of ν_e, ν_μ and ν_τ . To test whether the neutrino arriving in the detector is still a ν_μ or a linear combination, what one does is to allow it, for example, to impinge on a nucleus, as depicted in Figure 8(a), and see whether a μ is always produced or sometimes an e or τ . Such a procedure assumes of course that the W -boson couples always a μ to a ν_μ , or that, by time-reversal of Figure 8(a), the neutrino produced in the μ -nucleus collision of Figure 8(b) is always that particular linear combination of the mass eigenstates ν_i that we called ν_μ .

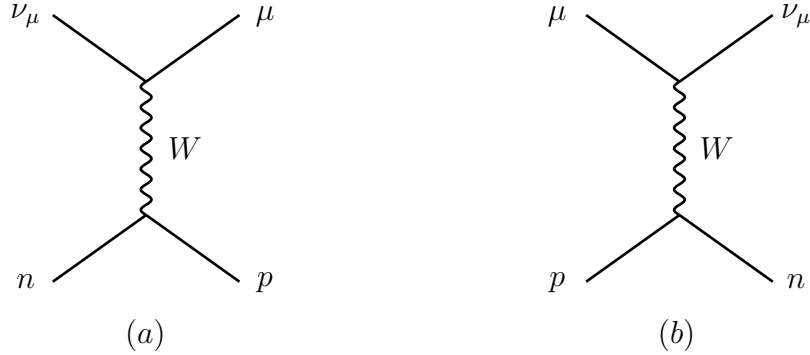


Figure 8: ν_μ -nucleus collision producing μ and the time-reversed process

The last assertion seems obvious until we start to entertain the idea that mass matrices rotate with changing scales. The decay process depicted in Figure 7 occurs at the μ -mass scale $\mu = m_\mu$, while the production process of Figure 8(b) occurs at a different scale depending on the energy. Given that the mass matrix rotates and thus have different orientations and therefore different eigenstates at different scales, can we be sure that the neutrino produced in the second reaction will always be the same linear combination of ν_i as that obtained from μ -decay?

This question can be unambiguously answered in the present framework

by repeating the calculation performed above for the reactions (44) and (45), namely by evaluating the Feynman digrams with loop insertions to the tree-diagrams in respectively Figures 7 and 8(b). This calculation has not been done. However, if we assume that the same result holds here for W -exchange as in reactions (44) and (45) for γ -exchange, then the answer to the above question is affirmative (up to order s/M_K^2), namely that the neutrino obtained from the reaction of Figure 8(b) is indeed the same as that obtained from μ -decay. In other words, the neutrino obtained from μ -decay impinging immediately on a nucleus, i.e. without allowing time for it to oscillate, will produce always a μ in the final state. This is of course the premises on which the experimental analysis is done and is usually taken as obvious, but in the case where the mass matrix rotates, it is an assertion which has to be demonstrated, and the above argument would now supply the answer. We note that the same question arises not just in the DSM but in principle in any scheme where the mass matrix rotates, including in particular the Standard Model as traditionally formulated [10, 11] although there, the rotation being much slower than in the DSM, it is not of as much practical significance.

A similar discussion can be extended to other processes to conclude, for example, that the ν_μ obtained from π -decay is indeed the same as that obtained from μ -decay (again up to order s/M_K^2) although the decays occur in principle at different scales. Also, by extending the argument to quarks, similar arguments would lead to the conclusion, for example, that the CKM mixing element V_{cb} as measured in the reaction $bp \rightarrow cn$ by exchanging a W will be the same (to order s/M_K^2) as the V_{cb} measured in b (i.e. D) decay and independent of scale.

7 Concluding Remarks

As in other attempts to explain fermion generations as a “horizontal symmetry” [18], the DSM scheme necessarily leads to flavour-violaton and has to guard against its excessive manifestation. A more obvious type of flavour-violation due to exchanges of gauge bosons associated with the gauged generation symmetry is easier to guard against since this is dependent on the gauge boson mass, usually to its fourth power, and so can conveniently be suppressed beyond any given experimental bound by assigning to the flavour-changing bosons a sufficiently high mass. This genre of flavour-violation in the DSM scheme, as mentioned above, has already been investigated in some detail [14, 23] and it was found that by choosing a mass-scale for the associ-

ated bosons of the order of 500 TeV, all existing bounds on flavour-violation can be satisfied. There is however in DSM another genre of flavour-violation which is potentially much more dangerous and which comes about because the scheme has the, as far as we know, unique feature of explaining both fermion mixing and fermion mass hierarchy as consequences of the rotating mass matrix. The flavour-violation of this genre can be large depending on the rate at which the mass matrix rotates, and since this rate is constrained in the scheme by the need to explain the observed magnitudes of fermion mixing and mass ratios between generations, there is no adjustable parameter available for tuning the amount of implied flavour-violation to escape experimental bounds. Indeed, it was found previously [11, 13, 12] that judging by kinematics alone, a mass matrix rotating at the rate the DSM requires can give flavour-violation of a size readily detectable by modern experiments of high sensitivity such as BaBar [24] and Belle [25] so that at one stage we thought we have here a make-or-break test for the DSM mechanism. Hence, the result in this paper that the implied flavour-violation is in fact much smaller, though in a sense a disappointment, is also a great relief, for otherwise if experiment finds no flavour-violation at the level predicted, there would in principle be no escape. As matters stand, however, the DSM is likely to survive tests along these lines for some time to come.

Although little flavour-violation is predicted in the DSM scheme, the conventional picture of flavour as a conserved quantity and of different flavour states as distinct objects is fundamentally changed. Flavour states rotate into one another so that flavour-violation naturally occurs, and though smaller than naively expected, flavour-violation is nevertheless present and in principle detectable. Indeed, the effects expected are similar in magnitude to flavour-changing neutral current effects [14, 23] and can possibly be observable soon by experiment under certain circumstances. Besides, in vector boson decays as studied in [12] where the B_L, B_R and L_L, L_R terms from respectively the wave function and vertex renormalization may not cancel exactly for lack of a Ward identity, flavour-violation need not be also of order s/M_K^2 and hence may be detectable already by current experiments [26, 27].

For the present moment, however, the result of this paper seems to allow schemes like the DSM to both “have the cake and eat it”, i.e. both to explain the sizeable fermion mixings and mass ratios between generations by a mass matrix rotating at appreciable speed, and at the same time to avoid contradiction with experiment as regards flavour-violation. Besides, as explained in the preceding section, it helps to resolve some conceptual difficulties con-

cerning the definition of mixing matrices and neutrino oscillations when the mass matrix rotates.

Within the DSM framework, the present paper represents also a certain technical step forwards in that previous studies of scale-dependent renormalization effects have been limited to only single-particle properties such as masses and mixing angles, but have now been extended to two-body properties observable only in collisions.

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